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Propagation of optical coherence lattices in the turbulent atmosphere

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We explore the propagation of recently introduced optical coherence lattices (OCLs) in the turbulent atmosphere. We show that the lattice intensity profile and the spatial degree of coherence will display periodicity reciprocity over long propagation distances even though the lattices are affected by the turbulence. The lattice periodicity reciprocity has been previously conjectured to be advantageous for free-space information transfer and optical communications. We then show how one can increase the distance over which the lattice periodicity reciprocity is preserved in the turbulent atmosphere by engineering input lattice beam parameters. We also show that the OCLs have scintillation indices lower than those of Gaussian beams.

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In recent decades, there has been a growing interest in generating periodic arrays of partially coherent beams, either in free space [1–10] or in inhomogeneous media [11]. This interest is motivated by a multitude of potential applications, chiefly to free-space optical communications or self-imaging in graded-index media. In this connection, a class of partially coherent beams with periodic coherence properties, the so-called optical coherence lattices (OCLs), was lately discovered [8], following a recently introduced complex Gaussian representation of statistical beams and pulses [12]; in addition, the OCL properties on free-space propagation have been explored [9]. In particular, the OCL degree of coherence at the source and their intensity distributions were shown to possess periodicity reciprocity: the initial periodic degree of coherence transfers its periodicity to the OCL intensity distribution on free-space paraxial propagation. This OCL feature was conjectured to make them attractive candidates for free-space information transfer and optical communications [9]. Specifically, the information encoded into a periodic source coherence pattern can be transmitted in free space and accessed in any receiver plane by interrogating an OCL intensity profile. More recently, experimental generation of OCLs carrying information was reported [10].

A potential caveat of the proposed protocol for long-distance communication channels, though, relates to the inevitable degradation suffered by OCLs due to the atmospheric turbulence. The study of optical beam propagation in the turbulent atmosphere is a venerable subject [13–29]. It is well established to date that as light beams propagate through the turbulence, they experience intensity fluctuations (scintillations), beam wander, and beam distortions caused by the refractive-index fluctuations in the atmosphere [27–29]. The latter will increase the bit error and, in general, scramble the transferred information encoded in a source beam. A variety of source beam configurations and propagation paths (e.g., optical fibers) or processing techniques (e.g., adaptive optics) have been proposed to mitigate undesirable effects imposed by the atmospheric turbulence [21,30,31]. The objective of this Letter is to examine turbulence effects on the OCL propagation and determine the conditions for periodicity reciprocity to survive the turbulence-induced degradation. In particular, we show how one can engineer OCL parameters at the source to optimize the information transfer capacity of OCLs in the turbulent atmosphere. We also calculate the OCL scintillation index in turbulence and demonstrate it to be lower than the Gaussian beam index.

We begin by considering the OCL source model, represented by the OCL cross-spectral density at the source as [8,9]

$$W(x_1,y_1,x_2,y_2,0)=\prod_{s=x,y}W(s_1,s_2,0)$$
$$=\prod_{s=x,y}\sum_{n_s=0}^{N}\frac{v_{n_s}}{\sqrt{\pi}}\exp\left[-\frac{s_1^2+s_2^2}{2\sigma_0^2}-\frac{2i\pi n_s(s_1-s_2)}{a\sigma_0}\right]. \quad (1)$$

Here n_s are non-negative integers; σ_0 is the beam waist size, serving as a characteristic transverse spatial scale hereafter; v_{n_s} is the power distribution of the pseudo-modes composing the lattice [8]; and a is a dimensionless lattice constant. It follows from Eq. (1) that the OCL reduces to the fundamental Gaussian beam when $N = 0$. The paraxial evolution of the OCL cross-spectral density in the turbulent atmosphere can be expressed as the generalized Huygens–Fresnel integral [15–19]

$$W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z)=\frac{1}{(\lambda z)^2}\exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}_1^2-\boldsymbol{\rho}_2^2)\right]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W(\boldsymbol{r}_1,\boldsymbol{r}_2,0)\times\exp\left[-\frac{ik}{2z}(\boldsymbol{r}_1^2-\boldsymbol{r}_2^2)\right]\exp\left[\frac{ik}{z}(\boldsymbol{\rho}_1\cdot\boldsymbol{r}_1-\boldsymbol{\rho}_2\cdot\boldsymbol{r}_2)\right]\times\langle\exp[\Psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)+\Psi^*(\boldsymbol{r}_2,\boldsymbol{\rho}_2)]\rangle\mathrm{d}^2\boldsymbol{r}_1\mathrm{d}^2\boldsymbol{r}_2, \quad (2)$$

where $\boldsymbol{r}_1 \equiv (x_1, y_1)$ and $\boldsymbol{r}_2 \equiv (x_2, y_2)$ are two arbitrary radius vectors in the source plane; and $k = 2\pi/\lambda$ is the wave number with λ being the wavelength, fixed at 1.55 μm henceforth. The asterisk denotes the complex conjugate; $\langle\ldots\rangle$ denotes the ensemble average. Suppose that the turbulence obeys the Kolmogorov statistics; by applying a quadratic approximation, the expression in the angular brackets in Eq. (2) can be expressed as [17]

$$\langle\exp[\Psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)+\Psi^*(\boldsymbol{r}_2,\boldsymbol{\rho}_2)]\rangle=\exp\left[-\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{\rho_0^2}+\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}{\rho_0^2}-\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2}{\rho_0^2}\right], \quad (3)$$

where $\rho_0 = (0.545C_n^2k^2z)^{-3/5}$ is the coherence length of a spherical wave propagating in turbulence, and C_n^2 is the refractive-index structure parameter.

Substituting from Eqs. (1) and (3) into Eq. (2), after a tedious integration, we obtain the following analytical expression for the OCL cross-spectral density in any transverse plane $Z = \text{const} > 0$:

$$W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,Z)=\prod_{s=x,y}\sum_{n_s=0}^{N}\frac{A(s_1,s_2)v_{n_s}}{\sqrt{\pi}\sigma(Z)}\exp\left\{\left[\frac{q^2\sigma_0^2Z}{R(Z)}-\frac{2q}{\sigma^2(Z)}\right](s_1-s_2)^2\right\}\times\exp\left[\frac{iq}{R(Z)}\frac{\pi n_s\sigma_0Z}{a}(s_1-s_2)\right]\exp\left[\frac{i}{2R(Z)}\left(\frac{1}{\sigma_0^2Z^2}-q\right)(s_2^2-s_1^2)\right]\times\exp\left[-\frac{i\pi n_s}{a\sigma_0}\frac{(s_1-s_2)}{\sigma^2(Z)}\right]\exp\left[-\frac{(s_1-\pi n_s\sigma_0Z/a)^2+(s_2-\pi n_s\sigma_0Z/a)^2}{2\sigma_0^2\sigma^2(Z)}\right], \quad (4)$$

where

$$A(s_1,s_2)=\exp\left[-\frac{i}{2\sigma_0^2Z}(s_1^2-s_2^2)-\frac{q}{2}(s_1-s_2)^2\right],$$
$$\sigma^2(Z)=Z^2(1+q\sigma_0^2)+1,\qquad R(Z)=\sigma^2(Z)/Z,$$
$$Z=\frac{z}{k\sigma_0^2},\qquad q=1/\rho_0^2. \quad (5)$$

Here Z is a dimensionless distance normalized by the Rayleigh range $z_R = k\sigma_0^2$ of a fully coherent Gaussian beam.

The intensity and spectral degrees of coherence are defined as [32]

$$\langle I(\boldsymbol{\rho},Z)\rangle = W(\boldsymbol{\rho},\boldsymbol{\rho},Z), \quad (6)$$

$$\mu(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,Z)=\frac{W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,Z)}{\sqrt{\langle I(\boldsymbol{\rho}_1,Z)\rangle\langle I(\boldsymbol{\rho}_2,Z)\rangle}}. \quad (7)$$

Equations (4), (6), and (7) are the key equations of this Letter. In the following, we show how the intensity distribution and the spectral degree of coherence change during the OCL propagation in the turbulent atmosphere. We note that there are uniformly or nonuniformly distributed OCLs, depending on the model weight distribution v_{n_s} [8,9]. In this Letter, we only show the uniform case, $v_{n_x} = v_{n_y} = v_0 = \text{const}$ with $1 \le n_{x,y} \le N$.

Figure 1 shows the intensity profile of a uniformly distributed OCL for different propagation distances Z in the turbulent atmosphere; the other parameters are the dimensionless lattice constant $a = 0.001$, the number of lattice lobes $N = 10$, and the refractive-index structure parameter $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$. One can infer from Fig. 1 that the periodicity reciprocity will be preserved over substantial distances, as long as $Z = 1$ in a weakly turbulent atmosphere. On the other hand, the periodicity reciprocity will break down at a certain distance after the propagation in the turbulent atmosphere, and the intensity distribution of an OCL becomes Gaussian, typical of any beam eventually succumbing to the turbulence. Thus, the initial Gaussian OCL intensity profile first splits and then recombines due to the turbulent atmosphere affects in a sharp contrast with the OCL evolution scenario in free space. Another aspect regarding the OCL behavior in the turbulent atmosphere relates to the magnitude spectral degree of coherence evolution, as evidenced in Fig. 2. The spectral degree of coherence will gradually attain a Gaussian shape but, thanks to the turbulence, the coherence length does not dramatically increase, unlike in the free-space case. Thus, the periodicity reciprocity properties of the OCL intensity and the degree of coherence can be utilized to transport information over some distances in the turbulent atmosphere, paving the way for optical communications through random media applications.

Further, we point out that the distance over which the OCL periodicity reciprocity holds decreases with the increase of the turbulence strength, as illustrated in Fig. 3. With the increase of the refractive-index structure parameter C_n^2 (the strength of the turbulence), the periodicity reciprocity of the intensity of the uniformly distributed OCL gradually disappears, and the OCL intensity distribution becomes Gaussian which is not shown in the figure. The same situation transpires for nonuniformly distributed OCLs.

Next, we show that by adjusting OCL parameters at the source, we can extend the distance over which the periodicity reciprocity holds, as is shown in Fig. 4. As one decreases the lattice constant a, the periodicity reciprocity persists over longer distances in the turbulent atmosphere, as is illustrated by the figure.

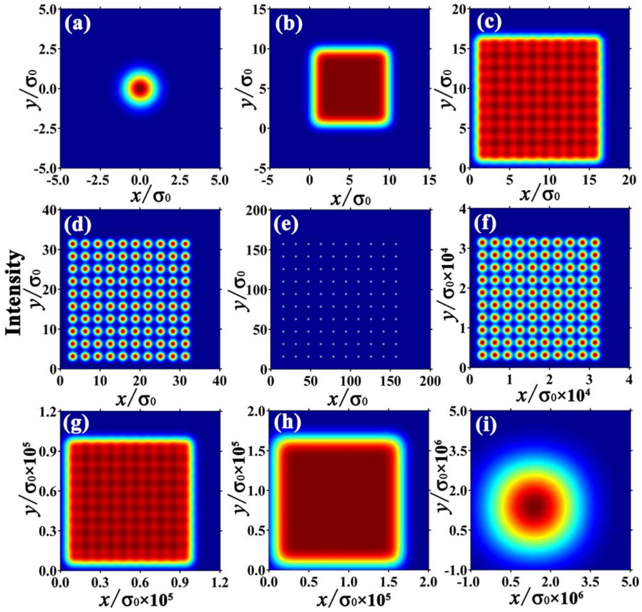

Fig. 1. Intensity distribution of a uniformly distributed OCL for different propagation distances (a) $Z = 0$; (b) $Z = 0.0005$; (c) $Z = 0.001$; (d) $Z = 0.005$; (e) $Z = 0.05$; (f) $Z = 1$; (g) $Z = 3$; (h) $Z = 5$; and (i) $Z = 80$ in the turbulent atmosphere with the lattice constant $a = 0.001$, the number of lattice lobes $N = 10$, and the refractive-index structure parameter $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$.

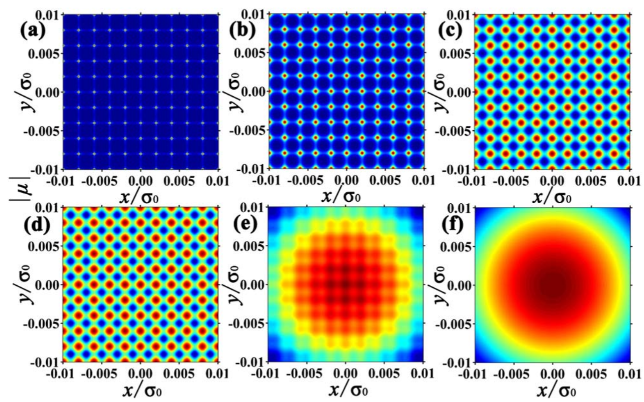

Fig. 2. Magnitude of the spectral degree of coherence of a uniformly distributed OCL for different propagation distances (a) $Z = 0$; (b) $Z = 0.0001$; (c) $Z = 0.0002$; (d) $Z = 0.0003$; (e) $Z = 0.0005$; and (f) $Z = 0.001$ in the turbulent atmosphere with the lattice constant $a = 0.001$, the number of lattice lobes $N = 10$, and the refractive-index structure parameter $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$.

Another important aspect of the OCL propagation through the turbulent atmosphere is that the OCL lattice structure can reduce the turbulence-induced scintillation, as compared to that of Gaussian beams. The OCL scintillation index is defined as [17,25–27]

$$m^2(\boldsymbol{\rho}, Z) = \frac{\langle I^2(\boldsymbol{\rho}, Z)\rangle}{\langle I(\boldsymbol{\rho}, Z)\rangle^2} - 1, \tag{8}$$

where $\langle I(\boldsymbol{\rho}, Z)\rangle$ and $\langle I^2(\boldsymbol{\rho}, Z)\rangle$ denote the average intensity and the average squared intensity. $\langle I(\boldsymbol{\rho}, Z)\rangle$ can be calculated

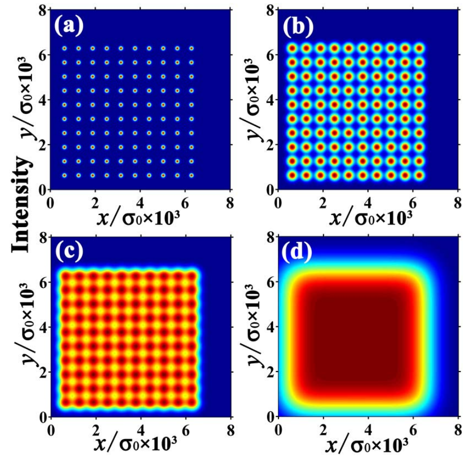

Fig. 3. Intensity distribution of a uniformly distributed OCL for different refractive-index structure parameters (a) $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$; (b) $C_n^2 = 5\times10^{-15}\ \mathrm{m}^{-2/3}$; (c) $C_n^2 = 10^{-14}\ \mathrm{m}^{-2/3}$; and (d) $C_n^2 = 5\times10^{-14}\ \mathrm{m}^{-2/3}$ with $a = 0.001$, $N = 10$, and $Z = 0.2$.

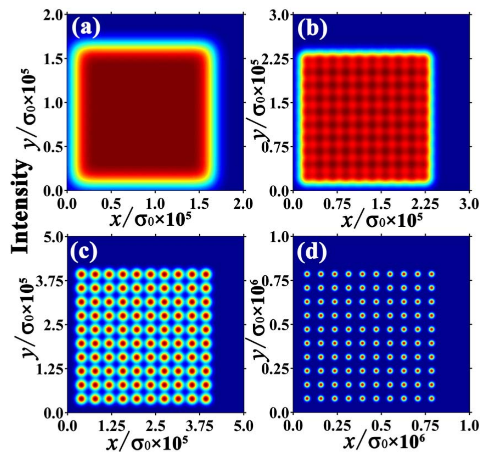

Fig. 4. Intensity distribution of a uniformly distributed OCL for different lattice constants (a) $a = 0.001$; (b) $a = 0.0007$; (c) $a = 0.0004$; and (d) $a = 0.0002$ with $Z = 5$, $N = 10$, and $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$.

from Eqs. (4) and (6) directly. $\langle I^2(\boldsymbol{\rho}, Z)\rangle$ can be calculated based on the fourth-order moment of the field in the turbulent atmosphere following the procedure outlined in [25–27].

The scintillation indices of OCLs in the beam centroid with different lattice constants a and lattice lobe numbers N are compared with those of the Gaussian beam ($N = 0$) in Fig. 5. In the calculation, we take the refractive-index structure parameter of the turbulence and the beam waist size to be $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$ and $\sigma_0 = 3$ cm, respectively. Our simulations show that by decreasing the lattice constant [Fig. 5(a)] or increasing the number of lobes [Fig. 5(b)], one can greatly reduce the OCL scintillation index. One can explain this phenomenon as follows. It can be implied from the coherent-mode representation of partially coherent beams [33] that spatial coherence of the beam is reduced with the number of contributing coherent modes. In our case, decreasing the lattice constant a or

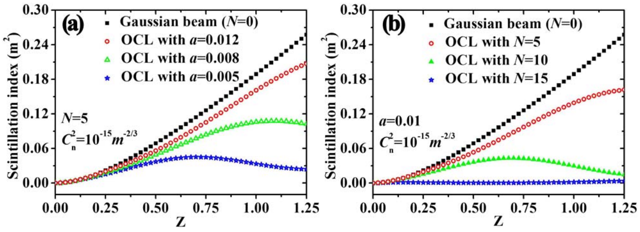

Fig. 5. Scintillation index of a uniformly distributed OCL in the beam centroid versus the propagation distance Z for (a) different lattice constants a and (b) different lattice lobes numbers N with $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$.

increasing the lattice lobe number N results in reduced spatial coherence of the OCL which, in turn, leads to the scintillation index reduction [25–28]. Although the blue plot in Fig. 5(b) looks nearly parallel to the horizontal axis, the scintillation index magnitude increases at long propagation distances [not shown in Fig. 5(b)]. It then follows that OCLs with a larger N have an advantage over those with a lower N, as well as over a Gaussian beam, at least, over sufficiently long propagation distances. Furthermore, the behavior of the OCL scintillation index on propagation is also affected by the OCL periodicity reciprocity. These findings provide further evidence of the OCL usefulness in free-space optical communications.

In summary, we have examined the OCL propagation in the turbulent atmosphere. We have shown that previously discovered periodicity reciprocity between the source degree of coherence and the propagated intensity profile of the lattices can be preserved in the turbulent atmosphere over a certain distance. We have also shown how one can engineer the OCL parameters at the source to enhance that distance. Finally, we have shown that the OCL scintillation index is reduced compared to that of the standard Gaussian beams propagating in the turbulent atmosphere. Our results will be useful for free-space information transfer and optical communications.

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